Derivatives 3, Chain Rule

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(trigger warning: these derivations are going to be some of the least rigorous things you have ever seen), Also I am doing this from scratch off the cuff so my notation for some of this earlier stuff is going to be pretty weird.

1 Introduction

Hello, another math derivation here. I do a lot of math derivations on my own, and I figured I should probably have a place to keep them, also I could use practice with LaTeXwrite ups.

The rule for these derivations is that I am not allowed any outside help and can only use other things I have derived. Still there are many spoilers so I try to avoid them, so far the only major spoilers I have had are knowing that the rule exists.

2 Goal

find

$$\frac{d}{dx}[f(g(x))]$$

3 The Derivation

1. Definition of the Derivative

$$\frac{d}{dx}[f(g(x))] = \lim_{dx \to 0} \frac{f(g(x+dx)) - f(g(x))}{dx}$$

2. dg(x) = g(x + dx) - g(x) (dg(x) + g(x) = g(x + dx)) (there is a limit for that dx but there is little space up here)

$$\lim_{dx \to 0} \frac{f(dg(x) + g(x)) - f(g(x))}{dx}$$

3. multiply by $\frac{dg(x)}{dg(x)}$

$$\lim_{dx \to 0} \frac{dg(x)}{dx} * \frac{f(g(x) + dg(x)) - f(g(x))}{dg(x)}$$

4. Separate the limits

$$\lim_{dx \to 0} \frac{dg(x)}{dx} * \lim_{dx \to 0} \frac{f(g(x) + dg(x)) - f(g(x))}{dg(x)}$$

5. Definition of the Derivative, $\lim_{dx \to 0} \frac{dg(x)}{dx} = g'(x)$

$$g'(x) * \lim_{dx \to 0} \frac{f(g(x) + dg(x)) - f(g(x))}{dg(x)}$$

6. $\lim_{dx \to 0} dg(x) = 0$ and since all instances of dx are within dg(x) changing the limit variable to dg(x) should not effect the relation of the rates of the top and bottom of the fraction. (not too much rigor but I need to take a real analysis class to know more)

$$g'(x) * \lim_{dg(x) \to 0} \frac{f(g(x) + dg(x)) - f(g(x))}{dg(x)}$$

7. Definition of the Derivative

$$g'(x) * f'(g(x))$$

8. Thus

$$\frac{d}{dx}[f(g(x))] = g'(x)f'(g(x))$$